

Mathesis Universalis and Information

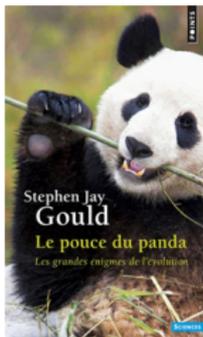
Hugues GENVRIN

August 2025

Prerequisites

Theory of Language

Location and Convention. The interpretation of natural language is the semantic language.

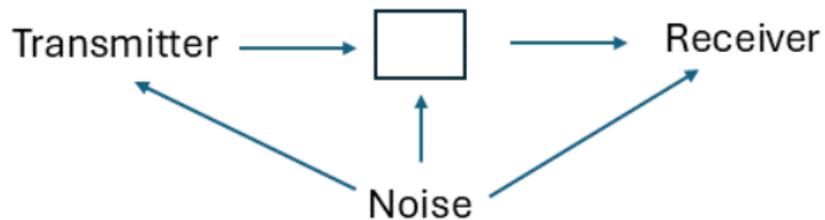


Natural language
Unit : sign
(Semiology)



Internal language
Unit : sense
(Semantic)

Pattern of a message



Formalize a sentence

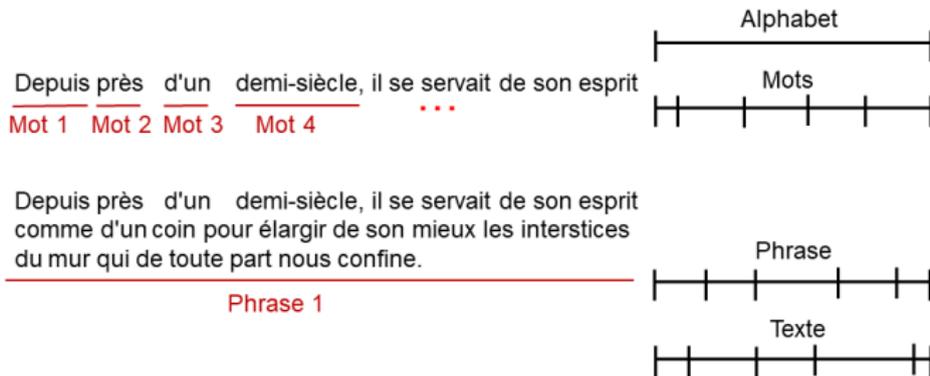


Figure – Formalize a sentence.

Speech Act : Ex « Je me marrie »

Extension to the internal language for the act of language



Austin
(1911-1960)

Example of internal language for an act : Pain.

Variable and functions

Function

One argument

$$y = f(x)$$

Two argument

$$z = f(x, y)$$

Natural language – Semiology

$$Sign = \phi(signified, signifier)$$

Unit



noise



Internal language - Semantic

$$Sense = \psi(denote, connotation)$$

Unit



noise



The symbol

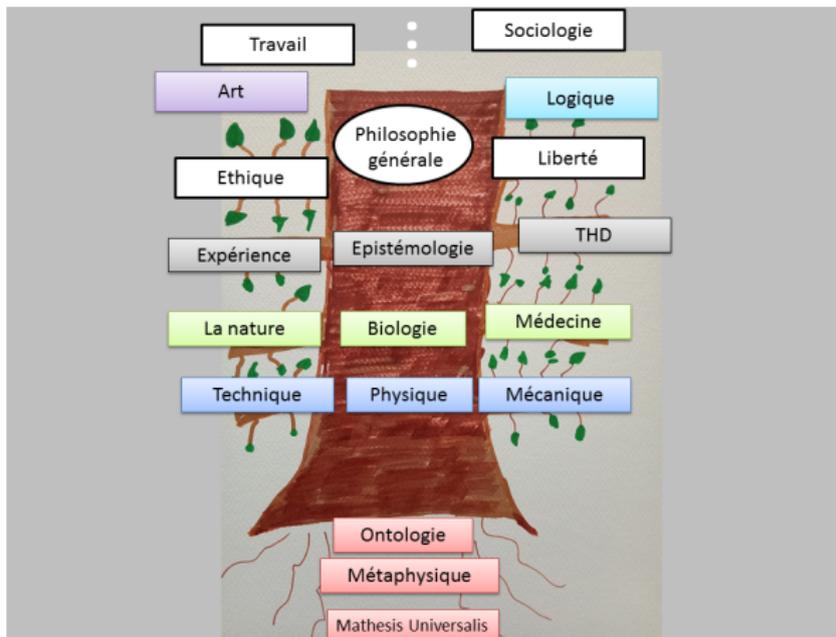
Definition

A symbol is representing the crystal growth of a sign or a sense. We pass from disorder (noise) to order.



MATHESIS UNIVERSALIS

Tree of knowledge



General philosophy

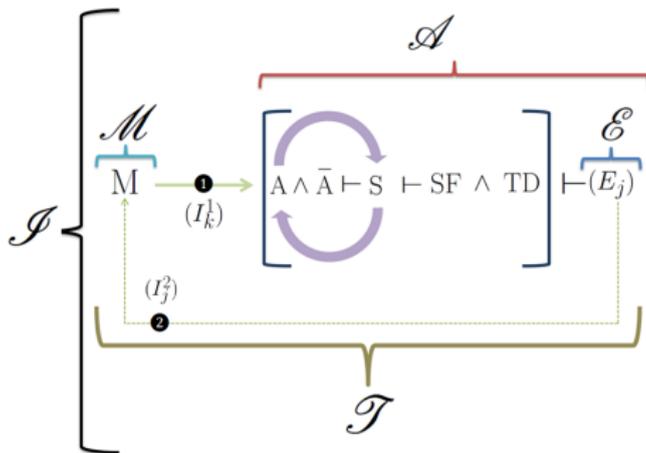
Hegelian dialectic and phenomenology



Phenomenology (Husserl)

Definition

An arkhectonic is an information system in linking relationship with a restriction of the world.



Etymologically, knowledge of the being.

Definition (Ontology(CNTRL))

Part of the philosophy which investigates the whole statement of the be.

Definition (Quantum, E.Kant (Critique of pure reason))

A quantum is an object endowed of quantity, it's allowed us to answer the question « How tall? »

Definition (Quantum of information : \mathcal{Q})

An information quantum is a concept endowed of magnitude, which brings informations on a system.

Klein's geometry

- 1 Let \mathcal{M} called a multiplicity either a set of elementary quantities or quantitas (Riemann).
- 2 We consider \mathcal{Q} as a quantum of information.
- 3 We call \mathcal{V} a manifold, which is transformaing a multiplicity \mathcal{M} under the action of a quantum of information : \mathcal{VRQ} .
- 4 Then, we definate a space as : $\mathcal{E} = \mathcal{VRQ}$.
- 5 Let (p_j) , properties taken on the manifold \mathcal{V} . They could be dynamic quantities.
- 6 Let (μ_k) a set of measures taken on properties.
- 7 For some transformations which are not altering the space, we note them (τ_j) . They are defining a principal group for the composition of transformations.
- 8 We will have other transformations which will let the space not invariant, altering some manifolds. We call them τ_j'' , the measure (μ_k) on properties (p_j) will be modified.
- 9 We call \mathcal{G} a geometry, which allowed us to realize measures on the properties (p_j) of the manifold \mathcal{V} .

We distinguish two restrictions of the space \mathcal{E} :

- The space of functioning (A.Sen) $\mathcal{E}_1^| = \mathcal{Q}_1 \mathcal{R}_1 \mathcal{V}_1$.
- The inner space $\mathcal{E}_2^| = \mathcal{Q}_2 \mathcal{R}_2 \mathcal{V}_2$, such as que $\mathcal{Q}_2 \subset \mathcal{V}_2$.

We are calling $\mathcal{E} = (\mathcal{Q}_1 \mathcal{R}_1 \mathcal{V}_1) \mathcal{R}_3 (\mathcal{Q}_2 \mathcal{R}_2 \mathcal{V}_2)$, such as $\mathcal{Q}_2 \subset \mathcal{V}_2$.

In this space \mathcal{E} , \mathcal{Q}_1 is a quantum of information, taken outside of the system. Whereas \mathcal{Q}_2 is a quantum of information taken in the part of the inner manifold.

We can advance that \mathcal{E} is defining a system which exists : $\tau(\exists)$ will design the existential transformations.

Example of natural selection

The theory of evolution from Darwin has been designed under four hypothesis :

- 1 Species are altering continuously.
- 2 Evolution is progressive, none abrupt.
- 3 The be wich are resemble are related.
- 4 The evolution is the result of natural selection.

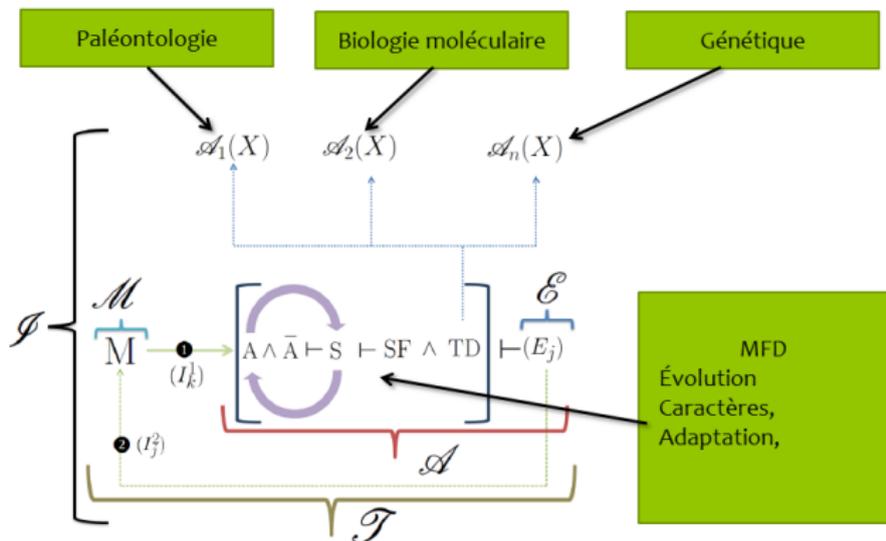
Evolution is showing us a phenotype evolution. We map to characters, predicates.

Definition (Predicate (CNTRL))

Quality, property, of a subject which is affirmed or rejected.

Evolution of those properties is showing us a dynamic ontology.

Dynamic Ontology



Definition (Metaphysic (CNTRL))

Fundamental part of philosophy thought which is treating the research of casuations, and the first principles.

Principe (de la classe entropique)

The entropic class, is the entelechia of all systems, could be entropic (\mathcal{H}) or neguentropic (\mathcal{N}).

We have $\mathcal{H} = \mathcal{H} > 0$ and $\mathcal{N} = \mathcal{H} \leq 0$.

This principle is a project (telos).

Generalization of the Fundamental Relation of Dynamic (GFRD)

By theorem, the GFRD is : $F_r = \dot{q}$.

This is the cause of dynamic of the systems.

As a theorem, we will apply this one to all formal systems, in every distinct areas of application.

Ex : The resultant force applied to sens is linking a domain and a co-domain, which are defining a semantic class and an entropic class.

Semantic information

We deduce the treatment channel of semantic information :

$$q(t) : (\text{Denotation} ; \text{Connotation}) \xrightarrow{F_r} q(t + \Delta t) : (\text{Denotation} ; \text{Connotation}) \\ t + \Delta t \dots \xrightarrow{F_r} q(t + n\Delta t) : (\text{Denotation} ; \text{Connotation})_{t + n\Delta t}.$$

We notice that : $\Delta q_{t+k\Delta t} = q_{t+k\Delta t} - q_{t+(k-1)\Delta t}$. The elementary variation taken on the temporal variation ($\Delta t \rightarrow 0$), are supporting by the action of a semantic resultant force : F_r .

So, we have a way of dynamic which is led by $F_r(t)$.

We will obtain a criterium of the end of the information treatment when

$$F_r(t + k\Delta t) = 0 \Leftrightarrow \Delta q(t + k\Delta t) = 0 \text{ with}$$

$q(t + k\Delta t) = q(t + (k - 1)\Delta t) = 0$, to have a safe equilibrium. Relaxation shoudn't be understand like an unsafe equilibrium

$$q(t + k\Delta t) = q(t + (k - 1)\Delta t) \neq 0.$$

Conversion between semantic and entropic class

$$\begin{aligned} \Psi_1 : \text{Semantic} &\longrightarrow \widehat{\mathcal{H}} \\ (\text{Denotation ; Connotation}) &\longmapsto (\mathcal{N} ; \mathcal{H}) \\ \text{q(semantic)} &\longmapsto \text{q}(\widehat{\mathcal{H}}). \end{aligned}$$

$$\begin{aligned} \Psi_1^{-1} : \widehat{\mathcal{H}} &\longrightarrow \text{Semantic} \\ (\mathcal{N} ; \mathcal{H}) &\longmapsto (\text{Denotation ; Connotation}). \end{aligned}$$

We will arise (erscheinung) senses associated to semantic manifolds.

$$\begin{array}{lcl} \Psi_1 : \text{Semiology} & \longrightarrow & \widehat{\mathcal{H}} \\ (\text{Signified}; \text{Signifiant}) & \longmapsto & (\mathcal{N}; \mathcal{H}). \end{array}$$

Entropic side :

$q(\widehat{\mathcal{H}}) : (\mathcal{N}; \mathcal{H}) \xrightarrow{F_r} q(\widehat{\mathcal{H}})(t + \Delta t) : (\mathcal{N}; \mathcal{H})_{t+\Delta t} \dots \xrightarrow{F_r} q(t + n\Delta t) : (\mathcal{N}; \mathcal{H})_{t+n\Delta t}$. By analogy, we obtain an entropic relaxation when $F_r(\widehat{\mathcal{H}}) = 0$. With a safe equilibrium $q(\widehat{\mathcal{H}})_{t+(k-1)\Delta t} = q(\widehat{\mathcal{H}})_{t+(k)\Delta t} = 0$.

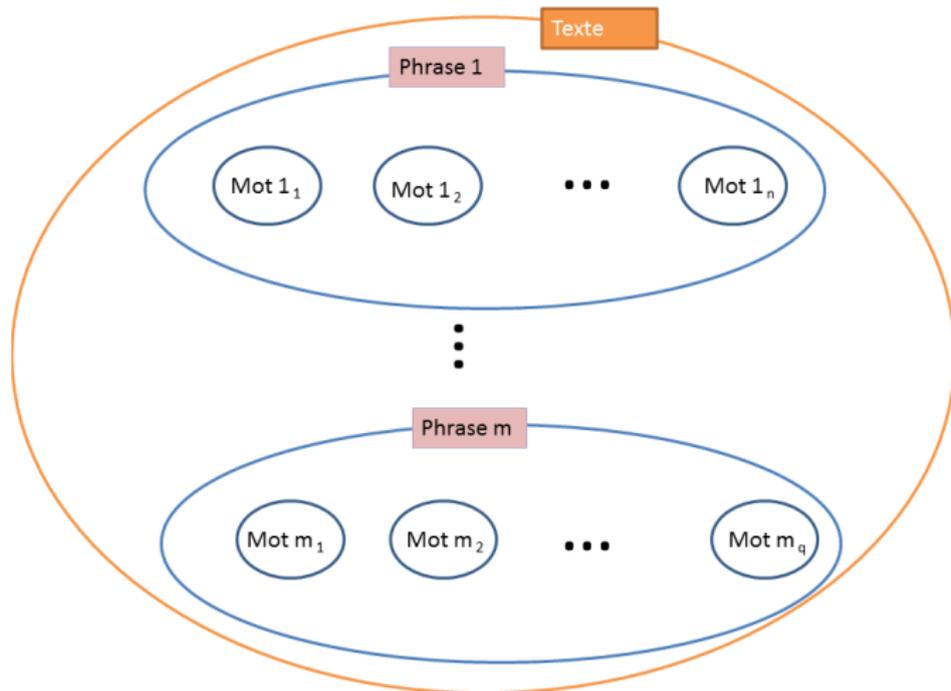
Relaxation shoudn't be understand like an unsafe equilibrium

$$q(\widehat{\mathcal{H}})_{t+(k-1)\Delta t} = q(\widehat{\mathcal{H}})_{t+(k)\Delta t} \neq 0.$$

Natural language

Negentropic composition

The natural language is a high-level of language.



Neguentropic composition

Le jeu du cadavre exquis

André Breton, Recueil pour un prélude, 1937

Princesse

déglutira

une petite mirabelle

gaiement

Figure – Jeu du cadavre exquis.

Neguentropic composition

Manchester and Cambridge studies

According to a research at Cambridge University, it doesn't matter in what order the letters in a word are, the only important thing is that the first and last letter be at the right place. The rest can be a total mess and you can still read it without problem. This is because the human mind does not read every letter by itself, but the word as a whole.

In this study, we find a word as proper-part or partial-part of the sentence.

Neguentropic composition

Shannon entropy

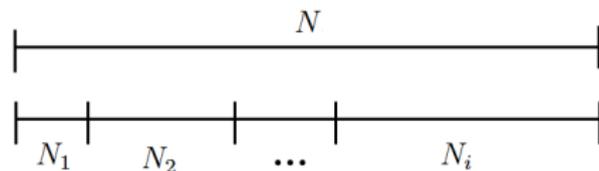


Figure – Partition of a set with N symbols.

$$\text{Let } \Delta_{\mathbb{E}} = \mathbb{E}[\mathcal{I}_{N_j}] - \mathbb{E}[\mathcal{I}_N] = \sum_{j=1}^i Pr_j \mathcal{I}_b(N_j) - \sum_{j=1}^i Pr_j \mathcal{I}_b(N) = \sum_{j=1}^i Pr_j \log_b \left(\frac{N_j}{N} \right) = \sum_{j=1}^i Pr_j \log_b (Pr_j).$$

Pour $b = e, \forall i, \sum_{j=1}^i Pr_j \ln(Pr_j) = -\mathcal{H}$. With \mathcal{H} as the Shannon entropy.

$-\mathcal{H} \leq 0 \Leftrightarrow -\mathcal{H} = \mathcal{N}_0$. So, $\Delta_{\mathbb{E}}$ is a neguentropic composition of the information.

Definition

Template (B.Stroustrup) A template is a class or a function that we parametrize with a set a types or values.

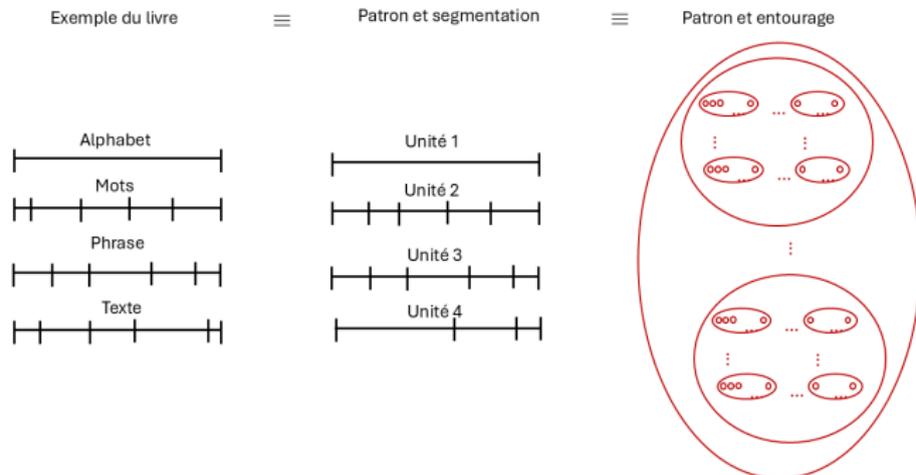


Figure – Template.

Application of the template

The template applied to a system with the units on an axial dimension, is showing the composition : Units 2 \circ Units 1, et Unit 3 \circ Units 2 \circ Units 1. They will define neguentropic composition.

Let :

$$\textcircled{1} \mathcal{N}(\text{Units } 2_k) \circ (\text{Units } 1_k) \geq \mathcal{N}(\text{Units } 1_k).$$

$$\textcircled{2} \mathcal{N}(\text{Unit } 3) \circ (\text{Units } 2_k) \circ (\text{Units } 1_k) \geq \mathcal{N}(\text{Units } 2_k \circ (\text{Units } 1_k)).$$

So : (1) $\Rightarrow \Delta \mathcal{N}_{2,1} \geq 0$ et (2) $\Rightarrow \Delta \mathcal{N}_{3,2} \geq 0$.

«J'ai une idée distincte du corps en tant qu'il est seulement une chose étendue et qui ne pense point, il est certain que ce moi, c'est à dire mon âme, par laquelle je suis ce que je suis, est entièrement et véritablement distincte de mon corps et qu'elle peut être ou exister sans lui ».

Méditations métaphysiques - *R.Descartes*

Example

Let a space \mathcal{E} which is defining measures $\mu(p_j)$ of a material system, while an existential transformation $\tau(\exists)$. More over than the measure, it's $\Delta\mu(p_j)$ which seems more interesting. $\text{SoF}_r(\hat{\mathcal{H}}) = \dot{p}_j$.

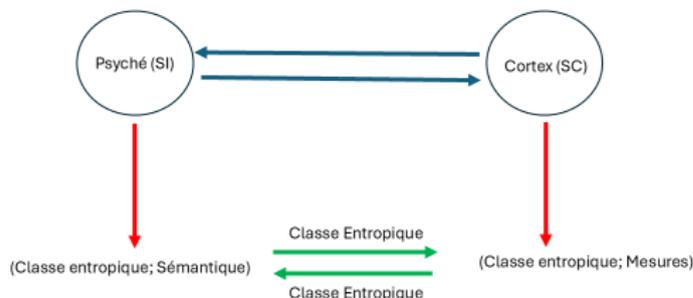


Figure – Separated Psyche and brain.

Quantitative Information

- 1 Typed language
 - 1 Multi-typed :
 - 1 Natural language (Grammar)
 - 2 Natural language (Semiology)
 - 2 Internal language (Semantic)
 - 3 Object language (Ex C++, VBA, Java)
- 2 None-typed language (Data flow)

Semiology : Treatment of information channel

On both sides : Informationnal (Semantic) and corporal

$$\begin{array}{lcl} \Psi_1 : \text{Semiology} & \longrightarrow & \widehat{\mathcal{H}} \\ (\text{Signified ; Signifiant}) & \longmapsto & (\mathcal{N} ; \mathcal{H}). \end{array}$$

Entropic side :

$q(\widehat{\mathcal{H}}) : (\mathcal{N} ; \mathcal{H}) \xrightarrow{F_r} q(\widehat{\mathcal{H}})(t + \Delta t) : (\mathcal{N} ; \mathcal{H})_{t+\Delta t} \dots \xrightarrow{F_r} q(t + n\Delta t) : (\mathcal{N} ; \mathcal{H})_{t+n\Delta t}$. By analogy, we obtain an entropic relaxation when $F_r(\widehat{\mathcal{H}}) = 0$. With a safe equilibrium $q(\widehat{\mathcal{H}})_{t+(k-1)\Delta t} = q(\widehat{\mathcal{H}})_{t+(k)\Delta t} = 0$.

Relaxation shouldn't be understand like an unsafe equilibrium

$$q(\widehat{\mathcal{H}})_{t+(k-1)\Delta t} = q(\widehat{\mathcal{H}})_{t+(k)\Delta t} \neq 0.$$

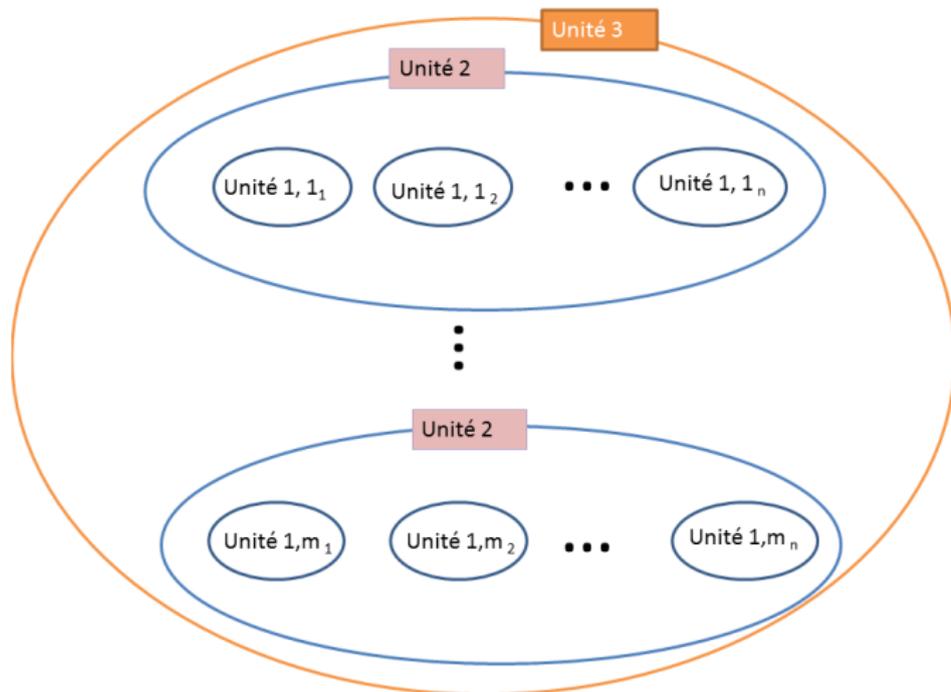


Figure – Template.



Figure – A.Rényi (1921-1970).

Definition (Quantity of Information)

$$\mathcal{I} = \log_2 N.$$

Where N is a quantity of symbols.

Example of application



Figure – Alphabetic symbols.

We need to code 27 symbols in base 2. So $2^4 = 16 < 27 < 2^5 = 32$.

Example of application

<i>Lettre</i>	<i>Codage</i>	<i>Lettre</i>	<i>Codage</i>	<i>Lettre</i>	<i>Codage</i>	<i>Codes inutilisés</i>
A	00000	J	1001	S	10010	11011
B	00001	K	1010	T	10011	11100
C	00010	L	1011	U	10100	11101
D	00011	M	1100	V	10101	11110
E	00100	N	1101	W	10110	11111
F	00101	O	1110	X	10111	
G	00110	P	1111	Y	11000	
H	00111	Q	10000	Z	11001	
I	01000	R	10001	Space	11010	

Figure – Encoding.

$\mathcal{I}_{\mathbb{N}}(\mathcal{A}) = 5$, because we need to code 27 symbols.

$\mathcal{I}_{\mathbb{R}}(\mathcal{A}) = \ln(27) = 4.75$,



Figure – Processor.



Figure – Dictionary.

Let's coding words \mathcal{M} with an octal, then $\mathcal{I}(\mathcal{M} \circ \mathcal{A}) = 40$.

Because $N = 2^{58} = 2^{40}$.

From $\mathcal{I}_{\mathbb{N}}$ to $\mathcal{I}_{\mathbb{R}}$

$$\mathcal{I}_{\mathbb{R}}(\mathcal{M} \circ \mathcal{A}) = (2^{4.75})^8 = 38.$$

$$\forall N \in \mathbb{R}_*^+$$

$$\mathcal{I}_{\mathbb{R}}^{10} = \log_{10}(2) \mathcal{I}_{\mathbb{R}}^2 = 0.30 \mathcal{I}_{\mathbb{R}}^2.$$

Rényi's quantity of information for the continuum

$$\mathcal{I} = n_\infty \Leftrightarrow N = 2^{n_\infty}.$$

Generalization of the Rényi's quantity of information

In \mathbb{N}

$$\mathcal{I}_{\mathbb{N}}^b \geq \log_b(N).$$

In \mathbb{R}

$$\mathcal{I}_{\mathbb{R}}^b = \log_b(N).$$

Shannon's entropy

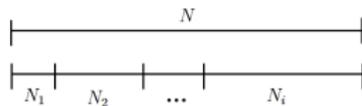


Figure – C.Shannon (1916-2001).

Theorem (Shannon's entropy)

$$\mathcal{H} = \sum_{i=1}^n Pr_i \ln\left(\frac{1}{Pr_i}\right).$$

Segmentation of data

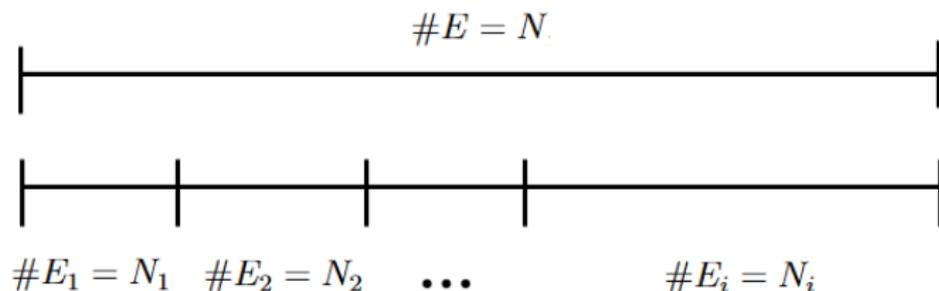


Figure – Segmentation of data.

$$\begin{aligned} X &: E \longrightarrow [0; 1] \\ &E_i \longmapsto X(E_i) = Pr_i = \frac{N_i}{N}. \\ X' &: E \longrightarrow \{1\} \\ &E \longmapsto X'(E) = 1. \end{aligned}$$

Shannon's entropy and Interpretation

$$\Delta_{\mathbb{E}} = \mathbb{E}[X] - \mathbb{E}[X'] = \sum_{i=1}^n Pr_i (\log_b \frac{N_i}{N}) = \sum_{i=1}^n Pr_i \log_b (Pr_i).$$

Let's choice $b = \exp$, $\mathcal{H} = \sum_{i=1}^n Pr_i \ln(\frac{1}{Pr_i})$.

Interpretation

With $\Delta_{\mathbb{E}}$, we quantify the variation of information between a state as a whole and an other segmented state.

Why the level of information is increasing ?

On one hand, $\Delta\mathbb{E}$ is an abstract function of the template. If it's apply to one class, it's valuable for every class. As in the natural language we have a \mathcal{N}_o , more other we have in all class, $\Delta^2\mathbb{E} < 0$ (except for the first step).

$$\Delta\mathbb{E} < 0 \wedge \Delta^2\mathbb{E} < 0 \Leftrightarrow \mathcal{N}_o.$$

On the other hand, Reny's information \mathcal{I} (denote) is a kind of disorder measure (connotation) of the math codes that we have to use to complete the symbol ($\# = N$). So $\Delta\mathbb{E} < 0 \wedge \Delta^2\mathbb{E} < 0$ implies the information is a negative measure of disorder, so nequentropic and this quantity is increasing.

Overview of Qualitative Information

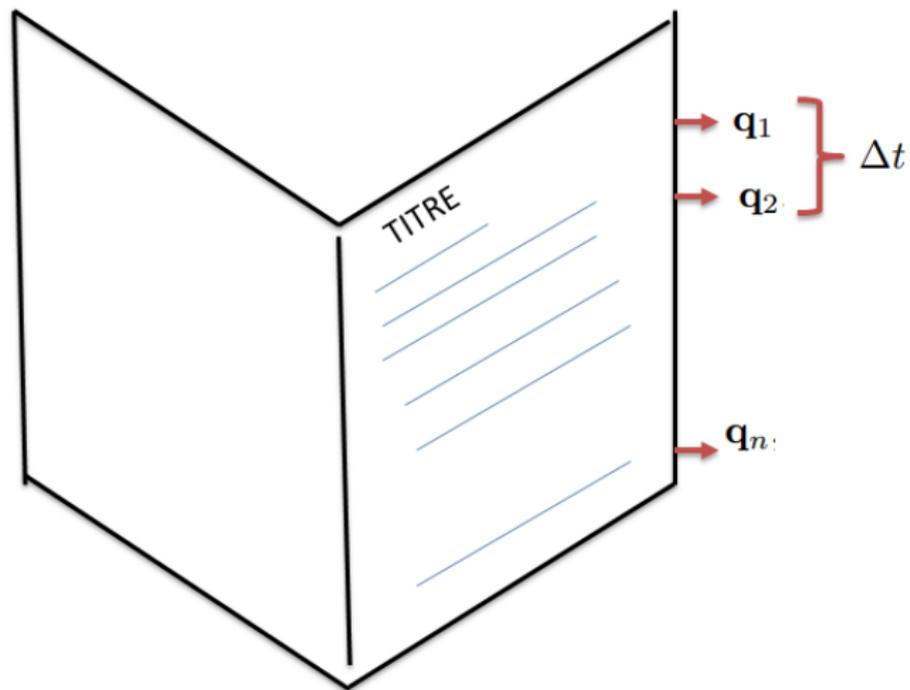


Figure – Discret approach

Semantic information

On both sides : Informational and corporal

We deduce the treatment channel of semantic information :

$$q(t) : (\text{Denotation}; \text{Connotation}) \xrightarrow{F_r} q(t + \Delta t) : (\text{Denotation}; \text{Connotation}) \\ t + \Delta t \dots \xrightarrow{F_r} q(t + n\Delta t) : (\text{Denotation}; \text{Connotation})_{t + n\Delta t}.$$

We notice that : $\Delta q_{t+k\Delta t} = q_{t+k\Delta t} - q_{t+(k-1)\Delta t}$. The elementary variation taken on the temporal variation ($\Delta t \rightarrow 0$), are supporting by the action of a semantic resultant force : F_r .

So, we have a way of dynamic which is led by $F_r(t)$. We will obtain a criterium of the end of the information treatment when

$$F_r(t + k\Delta t) = 0 \Leftrightarrow \Delta q(t + k\Delta t) = 0 \text{ with}$$

$q(t + k\Delta t) = q(t + (k - 1)\Delta t) = 0$, to have a safe equilibrium.

Conversion between semantic and entropic class

$$\begin{aligned}\Psi_1 : \text{Semantic} &\longrightarrow \widehat{\mathcal{H}} \\ (\text{Denotation ; Connotation}) &\longmapsto (\mathcal{N} ; \mathcal{H}) \\ \text{q(semantic)} &\longmapsto \text{q}(\widehat{\mathcal{H}}).\end{aligned}$$

$$\begin{aligned}\Psi_1^{-1} : \widehat{\mathcal{H}} &\longrightarrow \text{Semantic} \\ (\mathcal{N} ; \mathcal{H}) &\longmapsto (\text{Denotation ; Connotation}).\end{aligned}$$

We will arise (erscheinung) senses associated to semantic manifolds.

Topology and Template

Definition (Template (B.Stroustrup))

A template is a class or a function that we parametrize with a set a types or values.

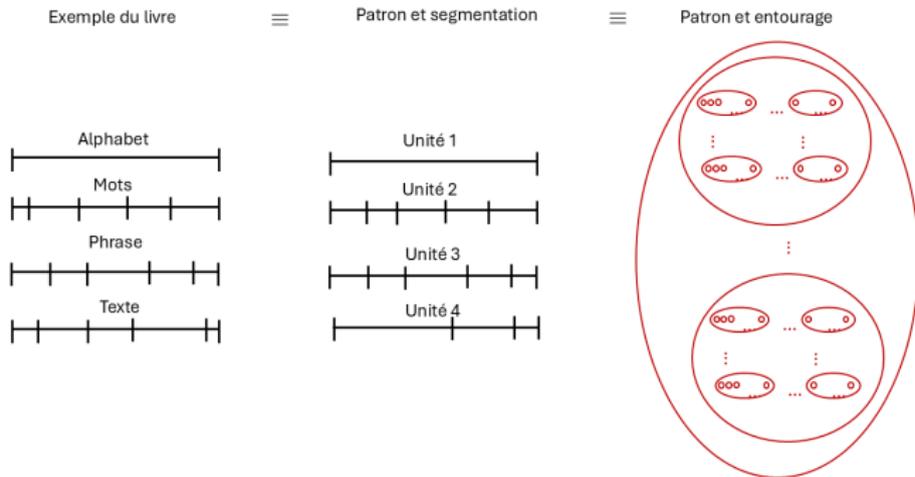


Figure – Template.

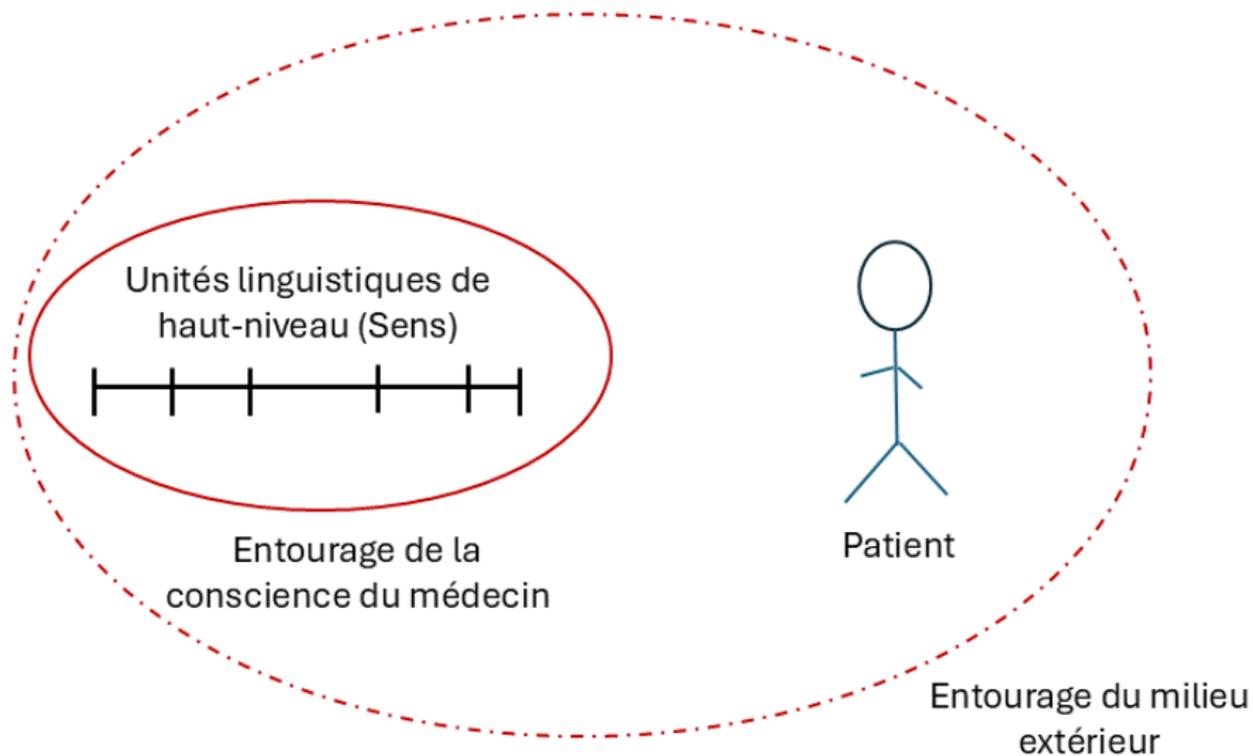
Semiologic and signify (factual) probability

A sign is a probability.

- Ex : a sign is not sure up to 100
- Ex : a sign is a probability to link toward a pathology in medecin.

We define a reverse probability called a bayes rule : $\Pr(\text{sign} \mid \text{signify})$.

Doctor's process



Probability function

$X : x_i \mapsto X(x_i)$ where x_i is a sign, this is the event. And

$Y : y_j = (\mathcal{R}_i x_i)_{1 \leq i \leq n}$.

$Y(\mathcal{R}_i x_i)_{1 \leq i \leq n}$ is defining a probability of signify.

$Y \circ X$ is a random function.

Hence $Y(x_i|y_i)$ is defining $Y(\text{sign}|\text{fact})$.

We have a recursive random process between adjacent segments, with signify and meta-signify.

Principle of none sufficient reason (Laplace)

Reverse Probability (Rule of Bayes).

- $\Delta \mathbb{E}_d = \sum_{i=1}^n Pr_i[\text{sign}_i|\text{fact}] \times \mathcal{I}(\text{fact})$
- $\Delta \mathbb{E}_c = \sum_{j=1}^m Pr_j[\text{fact}|\text{sign}_j] \times \mathcal{I}(\text{sign}_j)$
- $\Delta \mathbb{E}_d^i = Pr_i[\text{sign}_i|\text{fact}] \times \mathcal{I}(\text{fact})$
- $\Delta \mathbb{E}_c^j = Pr_j[\text{fact}|\text{sign}_j] \times \mathcal{I}(\text{sign}_j)$

The sign and the signify will denote an connote a sense in term of esperence. The esperence is semantic.

Mechanism in all life system

Biochimie is a melt of topological shapes

Cyanobactérie (Algue bleue)

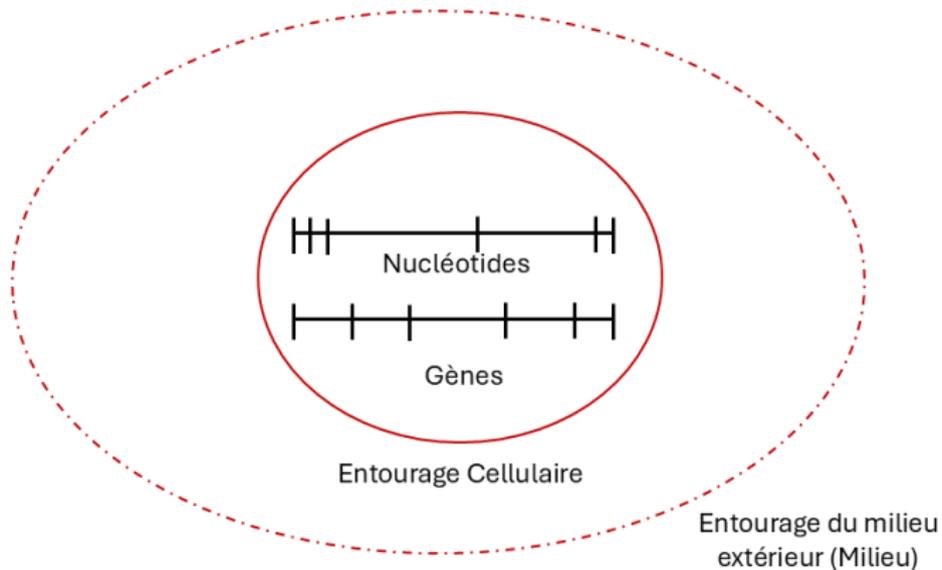


Figure – Cyanobactérie (Algue bleue).

$\widehat{\mathcal{H}}_c(sem)$ and $\widehat{\mathcal{H}}_d(sem)$.

Template language (abstraction) $\mathcal{R} \equiv SI \mathcal{R} \equiv SC$.

We observe that $\frac{\Delta q(sem)}{\Delta t} = F_r(SI) \equiv F_r(SC)$.

Semantic motor with a linkage between mathesis universalis and theory of information.

- ① Around the neguentropic composition :
 - Relationship between quantitative and qualitative information.
 - Margin \Leftrightarrow Consequence on the qualitative behaviour of the system.
- ② Consciousness act from the template, similar to speech of act from Austin, to the internal language.