

Logique

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Conjugate of privation

$$\begin{aligned}
 \overline{(A \Leftrightarrow B) \setminus (A \Rightarrow B)} &= ((\overline{A \Rightarrow B}) \wedge (\overline{A \Leftarrow B})) \setminus (\overline{A \Rightarrow B}) \\
 &= ((\overline{A \Leftarrow B}) \vee (\overline{A \Rightarrow B})) \setminus (\overline{A \Leftarrow B}). \quad (1) \\
 \overline{(A \Leftrightarrow B) \setminus (A \Rightarrow B)} &= \overline{A \Leftarrow B} \\
 &= \overline{A} \Rightarrow \overline{B}.
 \end{aligned}$$

$$\overline{\setminus} = \setminus$$

Conjugate of imply symbol

Relation of equivalence

Let $\mathcal{R} = \Leftrightarrow$ with the universe $\Omega_{\mathcal{R}} = \{0, 1\}$ such as $\bar{0}\mathcal{R}1$ et $\bar{1}\mathcal{R}0$,
où \bar{x} designates the conjugate of x .

1. $0\mathcal{R}0$ et $1\mathcal{R}1$, thus \mathcal{R} is reflexive.
2. For the reason that $0\mathcal{R}1$ and $1\mathcal{R}0$, so \mathcal{R} is symmetric.
3. If $a\mathcal{R}b$ et $b\mathcal{R}c$, then $a\mathcal{R}c$. Per example $0\mathcal{R}0$, et $0\mathcal{R}\bar{1} \Rightarrow 0\mathcal{R}0$. We have only two elements in $\Omega_{\mathcal{R}}$, thus the relation is obvious.

We deduce that \mathcal{R} is an equivalence relation : $\mathcal{R} \equiv$.

Conjugate of imply symbol

Conjugate of equivalence symbol

We notice that :

$$\overline{a \mathcal{R}_{\equiv} b} = \bar{a} \overline{\mathcal{R}_{\equiv}} \bar{b} \quad (2)$$

Hence $(\bar{0} \overline{\mathcal{R}_{\equiv}} \bar{0}) = (1 \overline{\mathcal{R}_{\equiv}} 1)$, such that $1 \mathcal{R} 1$. Then, it's involving that $\mathcal{R}_{\equiv} = \overline{\mathcal{R}_{\equiv}}$, which is equivalent to write : $\Leftrightarrow = \Leftrightarrow$.

Conjugate of imply symbol

Let $\Omega' = \{\Leftarrow, \Rightarrow\}$, then :

$$\begin{aligned} \Leftrightarrow &= (\Leftarrow \wedge \Rightarrow) \\ \text{is implying } \Leftrightarrow &= \overline{\Leftarrow \wedge \Rightarrow} \\ &= \Leftarrow \bar{\wedge} \Rightarrow \\ &= \Leftarrow \vee \Rightarrow \end{aligned} \tag{3}$$

As $\Leftrightarrow = \Leftarrow \vee \Rightarrow$, then : $\Leftrightarrow = \Leftarrow \vee \Rightarrow$.

Thus :

$$\begin{aligned} \Leftrightarrow &= (\Leftarrow \wedge \Rightarrow) \\ &= (\Leftarrow \bar{\wedge} \Rightarrow) \end{aligned} \tag{4}$$

cause $(\Leftarrow \wedge \Rightarrow) = (\Leftarrow \vee \Rightarrow)$

Consequently : for a none exclusive either, let $\Leftarrow = \Leftarrow \wedge \Rightarrow = \Rightarrow$;

or $\Leftarrow = \Rightarrow \wedge \Rightarrow = \Leftarrow$.

Conjugate of imply symbol

Order relation

Let $\mathcal{O}_{\Rightarrow}$ et \mathcal{O}_{\Leftarrow} , two relations on the universe $\Omega' = \{-1, 0, 1\}$.
With $\bar{0} = 0$, $\bar{1} = -1$.

1. $0\mathcal{O}_{\Rightarrow}0, -1\mathcal{O}_{\Rightarrow} -1$ et $1\mathcal{O}_{\Rightarrow}1$. Thus $\mathcal{O}_{\Rightarrow}$ is reflexive.
2. If $a\mathcal{O}_{\Rightarrow}b$ and $a\mathcal{O}_{\Leftarrow}b$, hence $b\mathcal{O}_{\Rightarrow}a$, thus $a \Leftrightarrow b$. Hence, the relation is anti-symmetric.
3. On another hand, $-1\mathcal{O}_{\Rightarrow}0$ and $0\mathcal{O}_{\Rightarrow}1$ which is involving $-1\mathcal{O}_{\Rightarrow}1$. So the relation $\mathcal{O}_{\Rightarrow}$ is transitive.

We deduce $\mathcal{O}_{\Rightarrow}$ is an order relation (generic, so un-strict). By an analogous reasoning, we have been to believe that \mathcal{O}_{\Leftarrow} is too an order relation.

Conjugate of imply symbol

In term of symbol

We have $\mathcal{O}_{\Rightarrow}$ et \mathcal{O}_{\Leftarrow} which are conjugates, in term of symbol :

$$\begin{cases} \mathcal{O}_{\Rightarrow} &= \overline{\mathcal{O}_{\Leftarrow}} = \mathcal{O}_{\Leftarrow\Leftarrow} \\ \mathcal{O}_{\Leftarrow} &= \overline{\mathcal{O}_{\Rightarrow}} = \mathcal{O}_{\Rightarrow\Rightarrow}. \end{cases}$$

Hence, in terms of connectors symbols :

$$\begin{cases} \Rightarrow &= \Leftarrow\Leftarrow \\ \Leftarrow &= \Rightarrow\Rightarrow. \end{cases}$$

Conjugate of imply symbol

$\Leftarrow \Rightarrow$ **et** $\Rightarrow \Leftarrow$

Contraposition

$A \Rightarrow B$, implies $\overline{A \Rightarrow B} = \overline{A} \Leftarrow \overline{B}$.

Theorem

$$A \Rightarrow B \Leftrightarrow \overline{B} \Rightarrow \overline{A}.$$

Corollaire

If $A \Rightarrow B = V$, then $\overline{A \Rightarrow B} = F \Leftrightarrow \overline{A} \Leftarrow \overline{B} = F$.

Corollaire

If $A \Rightarrow B = F$, then $\overline{A \Rightarrow B} = V \Leftrightarrow \overline{A} \Leftarrow \overline{B} = V$.

Set Logic

Let

$$\begin{aligned} \overline{A \setminus B} &= B \\ \overline{A \setminus B} &= \overline{A} \setminus \overline{B} \end{aligned} \tag{5}$$

If we have $\overline{\setminus} = \setminus$, then $\overline{A \setminus B} = \overline{A} \setminus \overline{B} = 0$.

As $B \neq 0$. We conclude :

$$\setminus \neq \overline{\setminus}.$$

A formule

Definition (Chaîne formelle)

A formal chain is a linguistic form. It's an expression of a formal language.

Definition (Formule : Fr)

A **formule Fr** is a broadly expression of a relationship, between a formal chain and arguments which will return values.

Let $Fr(x_1, \dots, x_n) = (y_1, \dots, y_m)$. Fr a boolean formule, n_∞ ou 2^{n_∞} arguments. $Fr|$ is a restriction, et par Fr^* , is a prolongement.

De l'infini potentiel vers l'infini actuel

$$\left\{ \begin{array}{l} \text{If } Fr(x \mapsto 2^{n_\infty}) = F, \text{ then } Fr^*(x = 2^{n_\infty}) = F \\ \text{If } Fr(x \mapsto n_\infty) = F, \text{ then } Fr^*(x = n_\infty) = F \end{array} \right.$$

Theorem

If a formule is wrong in the context of a potential infinite, it's wrong in the context of an actual infinite.

From actual infinite towards potential infinite

$$\left\{ \begin{array}{l} \text{If } Fr(x = 2^{n_\infty}) = V, \text{ then } Fr^|(x \mapsto 2^{n_\infty}) = V \\ \text{If } Fr(x = n_\infty) = V, \text{ then } Fr^|(x \mapsto n_\infty) = V \end{array} \right.$$

Let $Fr^|(x \mapsto 2^{n_\infty}) = F \Rightarrow Fr^{|\star}(x = 2^{n_\infty}) = F$.

Theorem

If a formula is true in the context of an actual infinite, it's true in the context of a potential infinite.

Formulation of the Cretan Paradox

The cretan philosopher Epimenides is at the origin of the cretan paradox, which has been revealed to the VI^o century before J-C. Here is how it was submitted.

1. All cretans are liars.
2. Epimenides the cretan tells that he is a liar.

We arrive to the conclusion : Epimenides lies and does not lie.
We will define the referents joined to the statements. The referent is a fonction wich returns syntactic items in natural language. Hence, for the statements system, we will have three referents.

1. r_1 =Cretan.
2. r_2 =Epimenide.
3. r_3 =Liar.

The last one is for the conclusion.

Those statements will be put in a matching relation by \mathcal{R} . We define by \bar{r}_3 the conjugate referent (opposition) as : not a liar. We also identify three states joined to the propositions and to the inferences.

1. $ST_1 = \forall r_1, r_1 \mathcal{R} r_3.$
2. $ST_2 = \exists r_2, r_2 \mathcal{R} r_1.$
3. $ST_3 = \exists r_2, r_2 \mathcal{R} r_3 \wedge r_2 \mathcal{R} \bar{r}_3.$

Undecidability

Definition (Undecidability)

A theory is called undecidable, if we can't show that we have either the statement p , or the statement \bar{p} .

We identify three kinds of undecidability :

1. The syntactic undecidability (Ladrière).
2. The power undecidability (the NNT in an experimental study).
3. The entropic undecidability (which contains disorder and heterogeneity).

Undecidability

As a deduction on ST_3 , we have Epimenide who is a liar, and Epimenide who is not a liar. We have $p \wedge \bar{p}$ which lead to have an entropic undecidability of second specie.

Definition

If we show a statement is p and \bar{p} , the law of contradiction leads to a second specie of entropic undecidability.